P.B. SIDDHARTHA COLLEGE OF ARTS & SCIENCE

Siddhartha Nagar, Vijayawada – 520 010
Reaccredited at 'A+' level by NAAC
Autonomous & ISO 9001:2015 Certified

Title of the Course: GRAPH THEORY
Semester : I

Course Code	23MA1T4	Course Delivery Method	Blended Mode
Credits	5	CIA Marks	30
No. of Lecture Hours / Week	5	Semester End Exam Marks	70
Total Number of Lecture Hours	75	Total Marks	100
Year of Introduction : 2023-2024	Year of offering : 2023-2024	Year of Revision: ----	Percentage of Revision :--

Course Objectives : To develop skills and to acquire knowledge on some basic concepts in connected graphs, Euler graphs, Hamiltonian graphs, Trees and Circuits, Planar graphs and Dual graphs etc.

Course Outcomes: After successful completion of this course, students will be able to

CO1: understand the properties directed graphs, Euler and Hamiltonian graphs. (PO1)

CO2: understand the properties of trees. (PO3)

CO3: illustrate the properties of cut sets and cut vertices. (PO4)

CO4: detect the planarity of a graph. (PO3)

CO5: illustrate the structure of a graph as a vector space. (PO1)

UNIT-I

Introduction: What is a Graph, Finite and Infinite graphs, Incidence and degree, Isolated Vertex, Pendant Vertex and Null Graph.

Paths and circuits: Isomorphism, Subgraphs , a puzzle with multi colored cubes. walks, Paths and Circuits, connected graphs, Disconnected graphs, Components, Euler graphs , Operations on graphs, More on Euler graphs, Hamiltonian paths and circuits, Travelling – Salesman Problem. (Chapters 1 and 2 of [1]).

UNIT-II

Trees and Fundamental Circuits: Trees , some properties of trees , pendant Vertices in a tree, distances and centers in a tree, rooted and binary trees, on Counting trees, spanning trees, fundamental circuits, finding all spanning trees of a graph , spanning trees in a weighted Graphs. (Chapter 3 of [1])

UNIT-III

Cut sets and Cut –vertices: Cut sets, Some Properties of a Cut Set, All cut sets in a Graph, Fundamental circuits and cut sets, connectivity and separability, network flows, 1-isomorphism, 2- isomorphism. (Chapter 4 of [1])

UNIT-IV

Planar and dual graphs: Combinatorial Vs Geometric graphs , Planer graphs, Kuratowski's two graphs , Different representations of a planar graph , Detection of planarity, Geometric dual. (Sections 1 to 6 of Chapter 5 of [1])

UNIT-V

Vector spaces of a graph: Sets with one operation, Sets with two operations, Modular arithmetic and Galois field, Vectors and Vector spaces, Vector space associated with a graph , Basis vectors of graph, circuits and cut-set sub spaces.
(Sections 1 to 7 of Chapter 6 of [1])

PRESCRIBED BOOK:

[1] " Graph theory with applications to Engineering and Computer Science", NARSINGH DEO, Prentice Hall of India Pvt., New Delhi,1993.

REFERENCE BOOK:

" Graph Theory with Applications", BONDY J.A AND U.S.R. MURTHY, North Holland,

Course has Focus on : Foundation

Websites of Interest:
1. www. nptel.ac.in
2. www.epgp.inflibnet.ac.in
3. www.ocw.mit.edu

P B SIDDHARTHA COLLEGE OF ARTS AND SCIENCE::VIJAYAWADA

(An Autonomous college in the jurisdiction of Krishna University)

M. Sc. Mathematics

First Semester

GRAPH THEORY –23MA1T4

Time: 3 Hours **Max. Marks : 70**

SECTION-A

Answer all questions **(5X4=20)**

1 a) Prove that the number of vertices of odd degree in a graph is always even. (CO1, L1)

(OR)

b) A connected graph G is an Euler graph if and only if it can be decomposed into circuits. (CO1, L1)

2 a) Prove that there is one and only one path between every pair of vertices in a tree. (CO2, L1)

(OR)

b) Prove that every tree has either one or two centers. (CO2, L1)

3 a) Show that every circuit has an even number of edges in common with any cut set. (CO3, L1)

(OR)

b) Define the edge connectivity of a graph. Show that the edge connectivity of a graph can never exceed the degree of the vertex with smallest degree in G. (CO3, L1)

4 a) Show that a graph can be embedded in the surface of a sphere if and only if it can be embedded in a plane. (CO4, L1)

(OR)

b) Prove that all duals of a planar graph are 2-isomorphic. (CO4, L1)

5 a) Prove that the set consisting of all the cut sets and the edge disjoint unions of cut sets in a graph G is an abelian group under the ring sum operation. (CO5, L2)

(OR)

b) Prove that the set of all circuit vectors in W_G forms a sub space of W_S. (CO5, L2)

SECTION-B

Answer all questions. All questions carry equal marks. **(5X10=50)**

6 a) If graph G has exactly two vertices of odd degree, then show that there must be a path joining these two vertices. (CO1, L2)

(OR)

b) Prove that a connected graph G is Euler graph if and only if all vertices of G are of even degree. (CO1, L2)

7 a) Prove that a tree with n vertices have n-1 edges. (CO2, L2)

(OR)

b) Show that every connected graph has at least one spanning tree. (CO2, L2)

8 a) Show that every cut set in a connected graph G must contains at least one branch of every spanning tree of G. (CO3, L2)

(OR)

b) Show that a vertex v in a connected graph G is a cut vertex if and only if there exists two vertices x and y in G such that every path between x and y passes through v. (CO3, L2)

9 a) Show that the complete graph with five vertices is non-planar. (CO4, L3)

(OR)

b) State and prove Euler's formula. (CO4, L3)

10 a) Prove that the ring sum of two circuits in a graph G is either a circuit or an edge disjoint unions of circuits. (CO5, L2)

(OR)

b) Prove that in a graph G, W_G is a vector space. (CO5, L2)

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